Part 7: Work, and Energy

University Physics V1 (Openstax): Chapters 7 and 8
Physics for Engineers & Scientists (Giancoli): Chapter 6

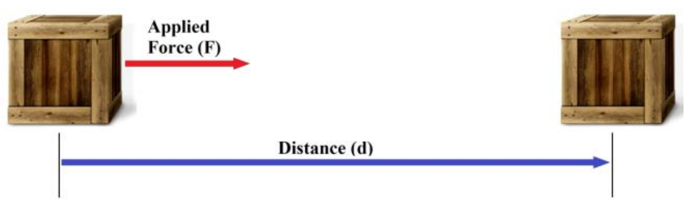
Work and Energy

- All motion and interactions can be understood in terms of energy and the exchange of energy.
- Work is derived from force (a vector), but work and energy are both scalar quantities (not vectors!)
- Many of the problems you have been working can be solved using an energy-based approach.
- In most cases, an energy-based approach to solving problems is preferable to other means.

In other words, don't go running back to kinematics on your homework!

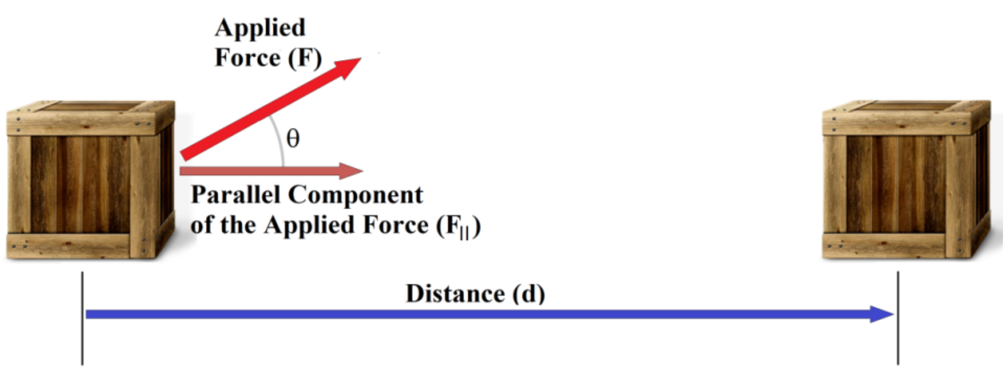
Work $W = \vec{F} \cdot \vec{d}$

- When an object moves a distance d in the direction of an applied force, the work done by that force is the product of the force and the distance.



$$W = F \cdot d$$

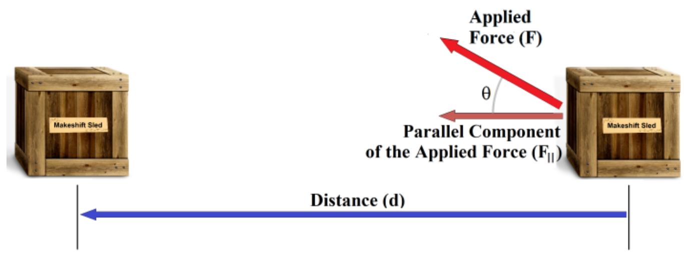
- The standard unit of work and energy the Joule. $1\ J = 1\ N \cdot m$
- When an object moves a distance d NOT in the direction of an applied force, the work done is the product of the parallel component of the force and the distance.



$$W = F_{\parallel} \cdot d = F \cdot d \cdot Cos(\theta) = \vec{F} \cdot \vec{d}$$

- This is also equivalent to multiply the applied force in full by the component of the distance in the direction of that force.

Example: Two horses pull a man on a makeshift sled. The man and the sled have a combined mass of 204 kg, and the force of friction between the sled and the ground is 700 N. When the horses pull the sled, each of the three chains has a tension of 396 N and makes an angle of 30.0° with respect to the horizontal as they pull the man a distance of 20.2 m. Determine A) the work done on the sled by one of the chains, B) the work done on the horses by one of the chains, and C) the work done on the sled by friction.



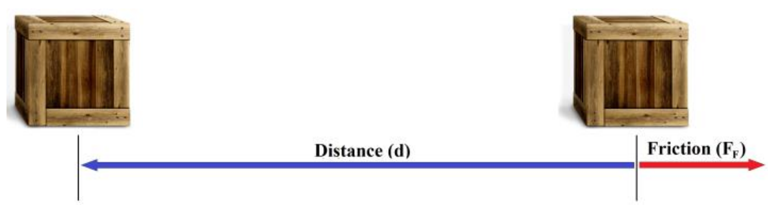
A) $W = F_{||} \cdot d = F \cdot d \cdot Cos(\theta) = (396\ N)(20.2\ m)Cos(30.0°) = 6.93\ kJ$

The positive sign on W indicates that the sled is gaining energy.

B) The angle between the force and the distance is now 180° ± θ

$W = F_{||} \cdot d = F \cdot d \cdot Cos(180° \pm \theta) = (396\ N)(20.2\ m)Cos(210.0°) = -6.93\ kJ$

The negative sign on W indicates that the horse is losing energy.
The energy lost by the horse is being given to the sled.



A) The angle between the force and the distance is now 180°

$W = F_{||} \cdot d = F \cdot d \cdot Cos(180°) = (700\ N)(20.2\ m)Cos(180°) = -14.14\ kJ$

The remaining energy absorbed by the sled is converted to motion of the sled.
Three chains each deliver 6.93 kJ and friction removes 14.14 kJ
Change in Energy = 3(6.93 kJ) – 14.14 kJ = 6.65 kJ

As friction always opposes motion, any work done by friction to a moving object will always be negative.

Example: How much work is done by a constant force, F = (3.26 N)$\hat{\mathbf{i}}$ + (5.67 N)$\hat{\mathbf{j}}$, as it acts on an object that moves from point P_1 = (1.23 m, 4.15 m) to point P_2 = (2.71 m, 3.85 m)?

$$\vec{d} = \Delta x\hat{\boldsymbol{\imath}} + \Delta y\hat{\boldsymbol{\jmath}} = (x_2 - x_1)\hat{\boldsymbol{\imath}} + (y_2 - y_1)\hat{\boldsymbol{\jmath}}$$

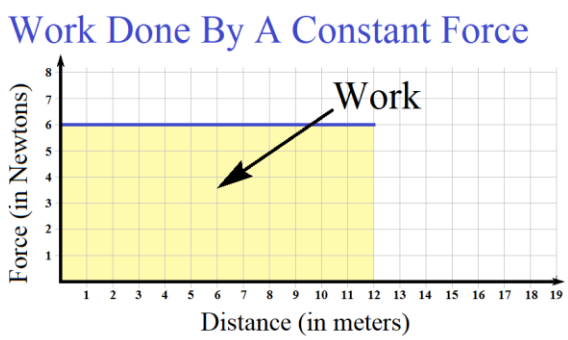
$$\vec{d} = (2.71\,m - 1.23m)\hat{\boldsymbol{\imath}} + (3.85\,m - 4.15m)\hat{\boldsymbol{\jmath}} = (1.48\,m)\hat{\boldsymbol{\imath}} + (-0.30\,m)\hat{\boldsymbol{\jmath}}$$

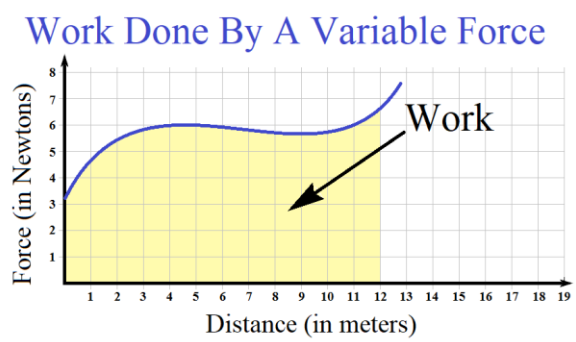
$$W = \vec{F}\cdot\vec{d} = \{(3.26\,N)\hat{\boldsymbol{\imath}} + (5.67\,N)\hat{\boldsymbol{\jmath}}\}\cdot\{(1.48\,m)\hat{\boldsymbol{\imath}} + (-0.30m)\hat{\boldsymbol{\jmath}}\}$$

$$W = (3.26\,N)(1.48\,m) + (5.67\,N)(-0.30m) = 3.12\,J$$

Work for Variable Forces

- For a constant force, the work may be considered to be the area created on a plot of force versus distance.
- The same holds true for a variable force. The work is the area under the curve when force is plotted against distance.





- In one dimensional motion: $W = \int_{x_1}^{x_2} F(x)dx$
- In higher dimensional motion: $W = \int_{P_1}^{P_2} \vec{F}\cdot d\vec{r}$
  - W is the work done
  - P_1 is the starting point
  - P_2 is the ending point
  - $\vec{F}$ is the force vector, which is a function of position.
  - $d\vec{r}$ is an infinitesimal displacement vector
  - $\vec{F}\cdot d\vec{r}$ is a vector dot-product

Example: An object moves from the origin to x = 1.53 m under the influence of a single force given by F(x) = (2.54 N/m^2)x^2 + (7.95 N/m)x + (16.95 N). Determine the work done by the force.

$$W = \int_{x_1}^{x_2} F(x)dx = \int_0^{1.53\,m} (\alpha x^2 + \beta x + \gamma)dx = \left\{\frac{\alpha}{3}x^3 + \frac{\beta}{2}x^2 + \gamma x\right\}_0^{1.53\,m}$$

$$W = \frac{\left(2.54\,\frac{N}{m^2}\right)}{3}(1.53\,m)^3 + \frac{\left(7.95\frac{N}{m}\right)}{2}(1.53\,m)^2 + (16.95\,N)(1.53\,m) = 38.3\,J$$

Example: An object moves from the origin to point P = (1.12 m, 1.74 m) under the influence of a single force F(x,y) = {(3.17 N/m)x}î + {(1.15 N/m^2)y^2 + (9.41 N)}ĵ. Determine the work done by the force.

$$W = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{r} = \int_{P_1}^{P_2} (F_x \hat{\imath} + F_y \hat{\jmath}) \cdot (dx\hat{\imath} + dy\hat{\jmath}) = \int_{x_1}^{x_2} F_x dx + \int_{y_1}^{y_2} F_y dy$$

$$W = \int_0^{1.12m} \alpha x dx + \int_0^{1.74\,m} \{\beta y^2 + \gamma\} dy = \left\{\frac{\alpha}{2}x^2\right\}_0^{1.12\,m} + \left\{\frac{\beta}{3}y^3 + \gamma y\right\}_0^{1.74\,m}$$

$$W = \frac{\left(3.17\frac{N}{m}\right)}{2}(1.12\,m)^2 + \frac{\left(1.15\frac{N}{m^2}\right)}{3}(1.74\,m)^3 + (9.41\,N)(1.74\,m) = 20.4\,J$$

Kinetic Energy $\quad KE = \frac{1}{2}mv^2$

- **Kinetic energy** is the energy an object possesses due to its motion.

$$W = \int_{x_1}^{x_2} F(x)dx = \int_{x_1}^{x_2} ma \cdot dx = \int_{x_1}^{x_2} m \cdot \frac{dv}{dt} \cdot dx = \int_{x_1}^{x_2} m \cdot \frac{dx}{dt} \cdot dv$$

Remember, dv and dx are just numbers and can be swapped, infinitesimally small, but numbers nonetheless.

$$W = \int_{v_1}^{v_2} mv \cdot dv = \left\{\frac{1}{2}mv^2\right\}_{v_1}^{v_2} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

- We define kinetic energy to be: $KE = \frac{1}{2}mv^2$
- This derivation suggests that any work done to an object results in a change in kinetic energy. This is valid when all the forces acting on an object are accounted for.

$$W = \Delta KE = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

- While this view is conceptually clearer, it is mathematically equivalent to the kinematics done previously (simply multiply an equation by ½m).

$$v^2 = v_0^2 + 2a(x - x_0) \qquad \frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 + ma(x - x_0)$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = ma(x - x_0) = Fd = W$$

Example: A 5.00 kg bald eagle is initially gliding horizontally at a speed of 11.3 m/s. It begins flapping its wings, generating a horizontal force of 19.6 N. How fast is the Eagle flying when it stops flapping its wings after a distance of 21.3 m??

$$W = \Delta KE \qquad Fd = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 \qquad 2Fd = mv^2 - mv_0^2 \qquad \frac{2Fd}{m} = v^2 - v_0^2$$

$$v^2 = v_0^2 + \frac{2Fd}{m} \qquad v = \sqrt{v_0^2 + \frac{2Fd}{m}} = \sqrt{\left(11.3\frac{m}{s}\right)^2 + \frac{2(19.6N)(21.3m)}{5.00\,kg}} = 17.2\frac{m}{s}$$

<u>Gravitational Potential Energy</u> $\mathbf{U_G = mgh}$

- When an object moves against the force of gravity, the work done lifting it becomes stored in the objects position (gravitational potential energy).

- When that object is released, the entirety of the stored energy is converted back into kinetic energy as it falls (assuming friction/wind resistance is not present).

- By definition, the potential energy of a force (U) differs by a negative sign from the work done moving against that force.

$$U = -W$$

- On the surface of the Earth, the gravitational force is effectively constant and equal to an object's weight (F = mg). Moving upward, the distance it moves would be the change in height. The force of gravity is directed opposite to the motion.

$$U_g = -W_g = -F_g \cdot d \cdot Cos(\theta) = -mg \cdot h \cdot Cos(180°) = mgh$$

- As we can choose to place our coordinate axis anywhere we desire, the gravitational potential energy of an object may vary with our choice of origin. Consequently, gravitational potential energy doesn't have an absolute value. Only the change in potential energy is relevant.

<u>Conservation of Energy</u>

- Energy is neither created nor destroyed. It only changes from one form to another.

- If work is done to an object due to external forces, it must be converted into either kinetic energy or potential energy (or some of both).

$$W_{NC} = \Delta KE + \Delta U$$

$$\Delta KE = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 \qquad \Delta U = mgy - mgy_0$$

  - The work done by non-conservative forces (W_{NC}) includes any energy added by applied forces as well as energy lost due to friction. Gravity is excluded as it is a conservative force and is covered under potential energy.

  - Conservative forces are those where the work done moving from one point to another does not depend on the path. Any work done by these forces is stored as potential energy (which might be returned later).

- It can be useful to rearrange this equation to find: $\boldsymbol{E_{init} + E_{added} = E_{final}}$

$$W_{NC} = \Delta KE + \Delta U = (KE_{final} - KE_{init}) + (U_{final} - U_{init})$$

$$W_{NC} = (KE_{final} + U_{final}) - (KE_{init} + U_{init}) = E_{final} - E_{init}$$

$$E_{init} + W_{NC} = E_{final} \qquad E_{init} + E_{added} = E_{final}$$

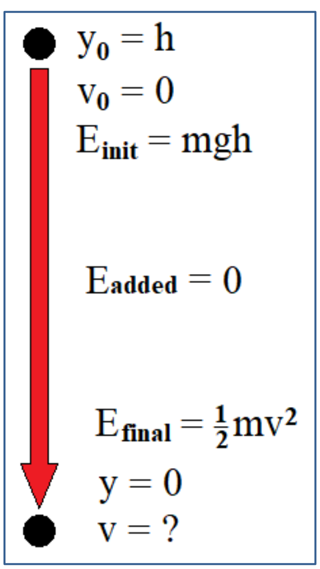
- The variable time does not show up directly in any form of energy. Any problem that includes motion without a collision and makes no mention of time is a good candidate to solve using conservation of energy.

- Energy lost due to friction is not actually lost, but rather it is transformed in heat (molecular vibrations).

Solving Problems with Conservation of Energy

- You only need to consider the initial and final states. Intermediate states are irrelevant.
- When you determine the initial and final energies, make sure to include every form of energy present.
    - Anything in motion will have kinetic energy.
    - Anything not at ground level (or where you decided to place y=0) will have gravitational potential energy.
- Make sure to traverse the path in between the initial and final positions to include anything that adds or removes energy to find E_{added} (W_{NC}).
    - Any friction forces will remove energy.
    - Other applied forces may add or subtract energy.
    - Gravity is accounted for with potential energy (don't include that with Eadded).

Example: A woman drops a small rock off a balcony 10.2 m above the ground. Assuming wind resistance is negligible, how fast is the rock moving just before it hits the ground?



We will set y = 0 to be at ground level.

Note: Setting y = 0 to occur at ground level is typically the most 'comfortable' thing to do. However, you may find that matching y = 0 to your lowest object is preferable. In this problem, both give the same origin.

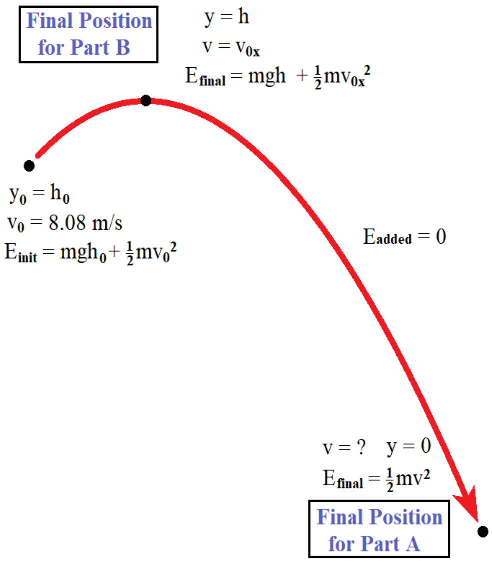
$$E_{init} = mgh \qquad E_{final} = \frac{1}{2}mv^2 \qquad E_{added} = 0$$

$$E_{init} + E_{added} = E_{final} \qquad mgh = \frac{1}{2}mv^2$$

$$gh = \frac{1}{2}v^2 \qquad 2gh = v^2$$

$$v = \sqrt{2gh} = \sqrt{2\left(9.80\frac{m}{s^2}\right)(10.2\ m)} = \sqrt{199.92\frac{m^2}{s^2}} = 14.1\frac{m}{s}$$

Example: A woman throws a small rock off a balcony 10.2 m above the ground. The initial velocity is 8.08 m/s and directed 30.0° above the horizon. Assume wind resistance is negligible. Determine A) the speed of the rock just before it hits the ground, and B) the maximum height of the rock.



We will set y = 0 to be at ground level.

Part A:

$$E_{init} = mgh + \frac{1}{2}mv_0^2 \qquad E_{final} = \frac{1}{2}mv^2 \qquad E_{added} = 0$$

$$E_{init} + E_{added} = E_{final} \qquad mgh + \frac{1}{2}mv_0^2 = \frac{1}{2}mv^2$$

$$gh + \frac{1}{2}v_0^2 = \frac{1}{2}v^2 \qquad 2gh + v_0^2 = v^2$$

$$v = \sqrt{2gh + v_0^2} = \sqrt{2\left(9.80\frac{m}{s^2}\right)(10.2\ m) + \left(8.08\frac{m}{s}\right)^2}$$

$$v = \sqrt{265.2064\frac{m^2}{s^2}} = 16.3\frac{m}{s}$$

Part B: $E_{init} = mgh_0 + \frac{1}{2}mv_0^2 \qquad E_{final} = mgh + \frac{1}{2}mv^2 \qquad E_{added} = 0$

$$v_y = 0 \qquad v = v_x = v_{0x} = v_0 \cdot Cos(\theta) = \left(8.08\frac{m}{s}\right)\cos(30.0°) = 6.9975\frac{m}{s}$$

$$E_{init} + E_{added} = E_{final} \qquad mgh_0 + \frac{1}{2}mv_0^2 = mgh + \frac{1}{2}mv^2 \qquad gh_0 + \frac{1}{2}v_0^2 = gh + \frac{1}{2}v^2$$

$$gh_0 + \frac{1}{2}v_0^2 - \frac{1}{2}v^2 = gh \qquad h_0 + \frac{v_0^2}{2g} - \frac{v^2}{2g} = h \qquad h = h_0 + \frac{v_0^2 - v^2}{2g}$$

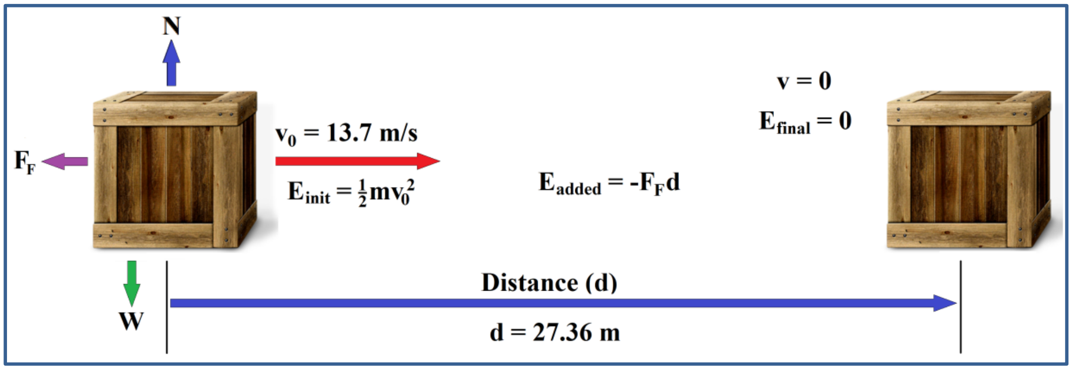
$$h = h_0 + \frac{v_0^2 - v^2}{2g} = 10.2\ m + \frac{\left(8.08\frac{m}{s}\right)^2 - \left(6.9975\frac{m}{s}\right)^2}{2\left(9.80\frac{m}{s^2}\right)} = 11.0\ m$$

Alternatively: $v_0^2 - v^2 = \left(v_{0x}^2 + v_{0y}^2\right) - v_{0x}^2 = v_{0y}^2$

$$v_{0y} = v_0 \cdot \sin(\theta) = \left(8.08\frac{m}{s}\right) \cdot \sin(30.0°) = 4.04\frac{m}{s}$$

$$h = h_0 + \frac{v_0^2 - v^2}{2g} = h_0 + \frac{v_{0y}^2}{2g} = 10.2\ m + \frac{\left(4.04\frac{m}{s}\right)^2}{2\left(9.80\frac{m}{s^2}\right)} = 11.0\ m$$

Example: Initially a crate is sliding on a horizontal surface at 13.7 m/s. The crate moves a distance of 27.36 m before coming to rest. Determine the coefficient of kinetic friction between the crate and the surface.



$$E_{init} = \frac{1}{2}mv_0^2 \qquad E_{final} = 0 \qquad E_{added} = -F_F d = -\mu_k N d = -\mu_k mgd$$

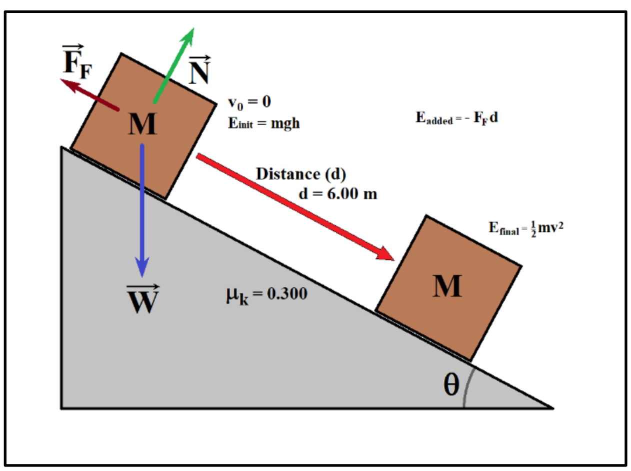
Note: As there are only two vertical forces (N and W) that must cancel, N=W=mg

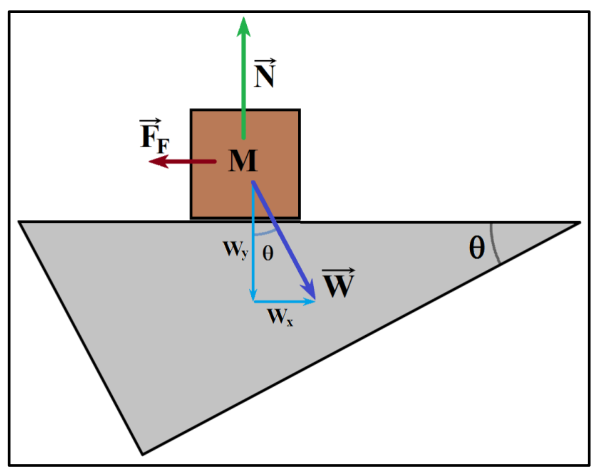
$$E_{init} + E_{added} = E_{final} \qquad \frac{1}{2}mv_0^2 - \mu_k mgd = 0 \qquad \frac{1}{2}mv_0^2 = \mu_k mgd \qquad \frac{1}{2}v_0^2 = \mu_k gd$$

$$\mu_k = \frac{v_0^2}{2gd} = \frac{\left(13.7\frac{m}{s}\right)^2}{2\left(9.80\frac{m}{s^2}\right)(27.36\,m)} = 0.350$$

Example: Initially a crate is at rest at the top of a 30.0° incline. The coefficient of kinetic friction between the crate and the surface is 0.300. How fast is the crate moving after sliding 6.00 m down the incline?

We will set y = 0 to be at ground level.





$$E_{init} = mgh = mgd \cdot \sin(30.0°) = \frac{1}{2}mgd \qquad E_{final} = \frac{1}{2}mv^2$$

The weight (W) does work, but this is not included in E_{added} as it is already covered with potential energy.

The normal force (N) doesn't do any work as it is perpendicular to the direction of motion.

$$E_{added} = -F_F d = -\mu_k N d = -\mu_k W_y d = -\mu_k mgd\cos\theta$$

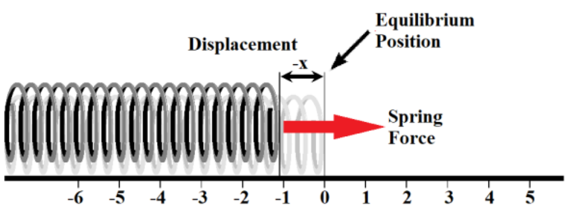
$$E_{init} + E_{added} = E_{final} \qquad \frac{1}{2}mgd - \mu_k mgd\cos\theta = \frac{1}{2}mv^2$$

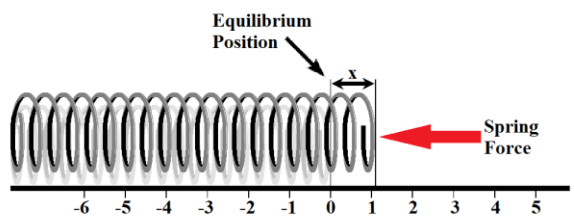
$$mgd - 2\mu_k mgd\cos\theta = mv^2 \qquad gd - 2\mu_k gd\cos\theta = v^2 \qquad v^2 = gd(1 - 2\mu_k\cos\theta)$$

$$v = \sqrt{gd(1 - 2\mu_k\cos\theta)} = \sqrt{\left(9.80\frac{m}{s^2}\right)(6.00\,m)\{1 - 2(0.300)\cos(30.0°)\}} = 5.31\frac{m}{s}$$

Springs and Hooke's Law

- Springs will naturally return to their initial (equilibrium) position after being stretched or compressed. A force that does this can be referred to as a **Restoring Force**.
- There are limits to how far springs can be stretched or compressed before losing their ability to stretch and becoming permanently deformed. This is called the **Elastic Limit** of the spring.
- With springs it is typical to use x as the change in length from equilibrium (equal to the displacement of the end) with it stretching into the positive axis and compressing into the negative axis.





- Hooke's Law: $F_x = -kx$
  - The negative sign indicates that the force points opposite the direction of displacement.
  - The spring constant, k, is only valid for a specific spring (not a universal constant).
  - Hooke's Law is only a first order linear approximation. Many springs will deviate from Hooke's Law before reaching the elastic limit.
  - Solid surfaces typically obey Hooke's law (albeit with very large spring constants). A very slight compression creates a large restoring force. This allows the normal force to take whatever value it needs to be.
  - Hooke's Law also applies to other objects that behave elastically.
- Elastic Potential Energy: $U_{sp} = \frac{1}{2}kx^2$
  - $U_{sp} = -W_{sp} = -\int_0^x F_{sp}(x)dx = -\int_0^x (-kx)dx = k\int_0^x (x)dx = k\left\{\frac{1}{2}x^2\right\}_0^x = \frac{1}{2}kx^2$
  - When using conservation of energy, it is preferable to include the elastic potential energy of springs rather than include it as work from an applied force (as you will just have to do this integral again).

Example: An archer pulls the bowstring back 42.0 cm and fires a 65.0 g arrow at 57.5 m/s. Determine the maximum height the arrow can reach if the archer pulls the bowstring back 50.0 cm.

For objects that obey Hooke's Law, the dynamic characteristics of that object have been reduced to a single quantity, the spring constant. If you don't know the spring constant of the object, you'll need to find that first.

$$E_{init} = \frac{1}{2}kx_1^2 \qquad E_{final} = \frac{1}{2}mv^2 \qquad E_{added} = 0$$

$$E_{init} + E_{added} = E_{final} \qquad \frac{1}{2}kx_1^2 = \frac{1}{2}mv^2 \qquad kx_1^2 = mv^2$$

$$k = \frac{mv^2}{x_1^2} = \frac{(0.0650\ kg)\left(57.5\frac{m}{s}\right)^2}{(0.420\ m)^2} = 1218.3\frac{N}{m}$$

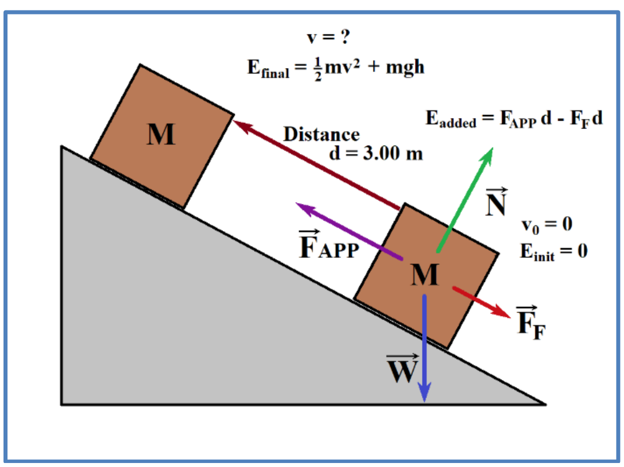
Now that we have the spring constant, we can look for the height.

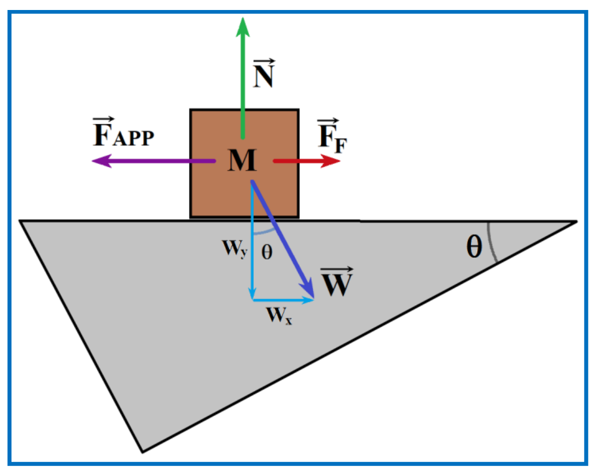
$$E_{init} = \frac{1}{2}kx_2^2 \qquad E_{final} = mgh \qquad E_{added} = 0$$

$$E_{init} + E_{added} = E_{final} \qquad \frac{1}{2}kx_2^2 = mgh$$

$$h = \frac{kx_2^2}{2mg} = \frac{\left(1218.3\frac{N}{m}\right)(0.500\ m)^2}{2(0.0650\ kg)\left(9.80\frac{m}{s^2}\right)} = 239\ m$$

Example: A worker pushes a crate up a 30.0° incline by delivering a 10.0 N force. The coefficient of kinetic friction between the crate and incline is 0.200. If the crate starts from rest and weighs 10.0 N, how fast is the crate moving after it traverses a distance of 3.00 m along the incline?





$$E_{init} = 0 \qquad E_{final} = \frac{1}{2}mv^2 + mgh$$

The weight (W) does work, but this is not included in E_{added} as it is already covered with potential energy.

The normal force (N) doesn't do any work as it is perpendicular to the direction of motion.

$$E_{added} = F_{App}d - F_F d \qquad F_F d = \mu_k N d = \mu_k W_y d = \mu_k mgd\cos\theta$$

$$E_{init} + E_{added} = E_{final} \qquad F_{App}d - \mu_k mgd\cos\theta = \frac{1}{2}mv^2 + mgh$$

$$F_{App}d - \mu_k mgd\cos\theta = \frac{1}{2}mv^2 + mgd\sin\theta \qquad F_{App}d - \mu_k mgd\cos\theta - mgd\sin\theta = \frac{1}{2}mv^2$$

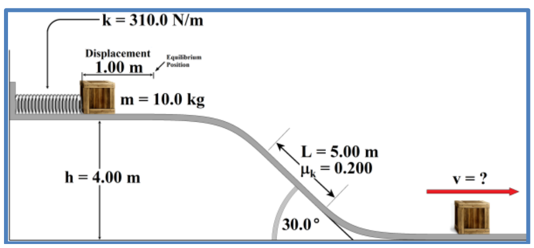
$$2F_{App}d - 2\mu_k mgd\cos\theta - 2mgd\sin\theta = mv^2$$

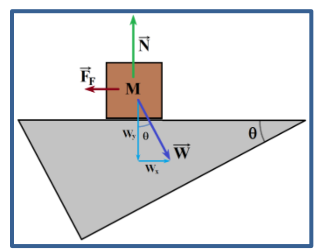
$$\frac{2F_{App}d}{m} - 2\mu_k gd\cos\theta - 2gd\sin\theta = v^2 \qquad \frac{2F_{App}gd}{W} - 2\mu_k gd\cos\theta - 2gd\sin\theta = v^2$$

$$2gd\left(\frac{F_{App}}{W} - \mu_k\cos\theta - \sin\theta\right) = v^2 \qquad v = \sqrt{2gd\left(\frac{F_{App}}{W} - \mu_k\cos\theta - \sin\theta\right)}$$

$$v = \sqrt{2\left(9.80\frac{m}{s^2}\right)(3.00\ m)\left(\frac{10.0\ N}{10.0\ N} - (0.200)\cos(30.0°) - \sin(30.0°)\right)} = 4.38\frac{m}{s}$$

Example: A distribution center has an interesting system for moving crates. A mechanism compresses a spring (k = 310.0 N/m) by 1.00 m. A 10.0 kg crate is then placed in front of the spring, which is then triggered sending the crate on its way. It falls a height of 4.00 m as it moves down a 30.0° incline before leveling off. The entire surface is frictionless except for a 5.00 m long stretch down the incline where the coefficient of friction is 0.200. How fast is the crate moving at the bottom of the incline?





$$E_{init} = \frac{1}{2}kx^2 + mgh \qquad E_{final} = \frac{1}{2}mv^2$$

$$E_{added} = -F_F d = -\mu_k N d = -\mu_k W_y d = -\mu_k mgd\cos\theta$$

$$E_{init} + E_{added} = E_{final} \qquad \frac{1}{2}kx^2 + mgh - \mu_k mgd\cos\theta = \frac{1}{2}mv^2$$

$$kx^2 + 2mgh - 2\mu_k mgd\cos\theta = mv^2 \qquad \frac{k}{m}x^2 + 2gh - 2\mu_k gd\cos\theta = v^2$$

$$v = \sqrt{\frac{k}{m}x^2 + 2gh - 2\mu_k gd\cos\theta}$$

$$v = \sqrt{\frac{310\frac{N}{m}}{10.0\ kg}(1.00m)^2 + 2\left(9.80\frac{m}{s^2}\right)(4.00\ m) - 2(0.200)\left(9.80\frac{m}{s^2}\right)(5.00\ m)\cos(30°)} = 9.61\ \text{m/s}$$

Power

- We define **Average Power** as: $P_{avg} = \frac{\Delta W}{\Delta t} = \frac{\Delta E}{\Delta t} = \frac{E_{final} - E_{init}}{t_{final} - t_{init}}$
- and **Instantaneous Power** as: $P = \frac{dW}{dt} = \frac{dE}{dt}$
- Power is defined using work, but the definition applies to any form of energy.
- The units of power are the Watt (W): 1 W = 1 J/s

- If the power is being delivered by a constant force: $P = Fv \qquad P = \frac{dW}{dt} = \frac{F \cdot dx}{dt} = F\frac{dx}{dt} = Fv$
- Kilowatt·Hour (kwh) is a unit of energy: $1\ kW \cdot Hr = (1000\ W)(3600\ s) = 3.6 \times 10^6\ J$

Example: A low-flying eagle with mass 4.50 kg increases its velocity from 11.3 m/s to 17.2 m/s over a 15.0 second time interval. Over the same time its altitude increases from 1.50 m to 7.75 m. What average power must the eagle's wings deliver to accomplish this?

$$E_{init} = \frac{1}{2}mv_0^2 + mgh_0 \qquad E_{final} = \frac{1}{2}mv^2 + mgh$$

$$P_{avg} = \frac{\Delta E}{\Delta t} = \frac{E_{final} - E_{init}}{\Delta t} = \frac{\frac{1}{2}mv^2 + mgh - \frac{1}{2}mv_0^2 - mgh_0}{\Delta t} = \frac{\frac{1}{2}m(v^2 - v_0^2) + mg(h - h_0)}{\Delta t}$$

$$P_{avg} = \frac{\frac{1}{2}(4.50\ kg)\left\{\left(17.2\frac{m}{s}\right)^2 - \left(11.3\frac{m}{s}\right)^2\right\} + (4.50\ kg)\left(9.80\frac{m}{s^2}\right)(7.75\ m - 1.50\ m)}{15.0\ s} = 43.6\ W$$

Example: The Space-X Falcon 9 rocket has a mass of 1.48×10^6 kg when loaded with payload destined for low-Earth orbit (leo). Its engines generate 22.8 MN (mega-Newtons) of thrust during their initial burn. When the first stage is jettisoned after 157 s, the rocket is going 1,839 m/s at an altitude of 70.4 km. What is the average power output of the engines?

Can we use 'U = mgh' at an altitude of 70.4 km? ...No, but we could use $F = GmM_E/r^2$ and integrate.

Is wind resistance negligible? ...No!

We must use work and not energy.

$$P_{avg} = \frac{\Delta W}{\Delta t} = \frac{Fd}{\Delta t} = \frac{(22.8 \times 10^6\ N)(70.4 \times 10^3\ m)}{157\ s} = 1.02 \times 10^{10} W$$

Note: The largest nuclear power plant in the US, the Palo Verde Nuclear power plant in Arizona has an output of 3.94×10^9 W and is a major source of electric power for the densely populated parts of Southern Arizona and Southern California, including Phoenix, Tucson, Los Angeles, and San Diego.

The Space-X Falcon 9 rocket delivers 2.5 times more power than the largest nuclear reactor in the country.